Charge transfer statistics in symmetric fractional edge-state Mach-Zehnder interferometer

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We have studied the zero-temperature statistics of charge transfer between the two edges of quantum Hall liquids with filling factors $\nu_{0,1} = 1/(2m_{0,1}+1)$ forming Mach-Zehnder interferometer. The known Bethe ansatz solution for symmetric interferometer is used to obtain the cumulant-generating function of charge at constant voltage V between the edges. Its low-V behavior can be interpreted in terms of electron tunneling, while its large-V asymptotics reproduces the m-state dynamics ($m \equiv 1+m_0+m_1$) of quasiparticles with fractional (for $m > 1$) charge and statistics. We also analyze the transition region between electrons and quasiparticles.

Mach-Zehnder interferometer (MZI) (Refs. [1]–[3]) based on the quantum Hall edge states in the regime of the fractional quantum Hall effect (FQHE), together with quantum antidots,[4,5] provides potentially useful tool[6–8] for the observation of fractional statistics of FQHE quasiparticles. In contrast to their fractional charge, which has been confirmed in experiments,[4,9,10] in particular, by measuring the Fano factor of a single tunneling contact, there are no generally accepted observations of the anyonic statistics of the quasiparticles. One of the difficulties is that, in typical experiments, the quasiparticles are produced at the edges, i.e., they emerge as continuum of gapless excitations described by a one-dimensional (1D) field theory.[11] In this theory, individual quasiparticles are tangible only asymptotically in a special limit. In the fractional edge-state MZI this limit occurs at *large* voltages, when the edge excitations are quantized by the strong tunneling potential, and the MZI Hamiltonian[8] is dual to the usual Hamiltonian of the electron tunneling.

In this work, we calculate the zero-temperature full counting statistics[12] of the charge transferred between the two edges of quantum Hall liquids forming Mach-Zehnder interferometer (Fig. 1). The high-voltage asymptotics of this statistics is explained by weak tunneling of fractionally charged quasiparticles of anyonic braiding statistics, which gives rise to the m-state dynamics of the MZI as a result of successive changes of the effective flux through it due to quasiparticle tunneling. Since the low-voltage charge transfer is still quantized in electrons, the found counting statistics reflects the effect of the anyonic phases on electron splitting into quasiparticles with increasing voltage V between the MZI edges. To characterize this crossover conveniently, we study the behavior of experimentally observable Fano factor of the MZI at different voltages. We show, in particular, that close to complete destructive interference in the interferometer, the quasiparticle statistics ensures that the MZI Fano factor retains its electronic value 1 for all voltages, including the quasiparticle tunneling range. Away from this regime, the Fano factor can reach the quasiparticle value $1/m$ at large voltages, and exhibits nonmonotonic voltage dependence with a minimum indicating the crossover between electrons and quasiparticles.

Our calculations start with the electronic model of MZI (Fig. 1) with two single-mode edges of filling factors $\nu_{0,1}$

$=1/(2m_{0,1}+1)$, and $m_0 \geq m_1 \geq 0$. In the standard bosonization approach,[11] electron operator ψ_l of the edge l is $\psi_l = (D/2\pi\nu_l)^{1/2}\xi_l \exp\{i[\phi_l(x,t)/\sqrt{\nu_l}+k_l x]\}$, where ϕ_l are the two chiral right-propagating bosonic fields, which satisfy the usual commutation relations $[\phi_l(x),\phi_p(0)]=i\pi\,\text{sgn}(x)\delta_{lp}$. The Majorana fermions ξ_l account for mutual statistics of electrons in the opposite edges, and D is a common energy cutoff of the edge modes. The Fermi momenta k_l correspond to the average electron density in the edges, while the density fluctuations are: $\rho_l(x,\tau)=(\sqrt{\nu_l}/2\pi)\partial_x\phi_l(x,\tau)$.

In the symmetric interferometer with equal times of propagation between the contacts along the two edges, it is convenient to introduce the tunneling field $\phi(x)$,

$$\phi(x) \equiv \left[\sqrt{\nu_1}\phi_0(x) - \sqrt{\nu_0}\phi_1(x)\right]/\sqrt{\nu_0+\nu_1}. \qquad (1)$$

The Lagrangian for electron tunneling at points $x=x_{1,2}$ is expressed then in terms of this field,

$$L_t = \sum_{j=1,2} (DU_j/\pi)\cos[\lambda\phi(x_j) + \kappa_j], \qquad (2)$$

where U_j and κ_j are the absolute values and the phases of the dimensionless tunneling amplitudes, the products of the Majorana fermions $\xi_1\xi_2$ were omitted, since they cancel out in each perturbative order due to charge conservation, and $\lambda \equiv (\nu_0^{-1}+\nu_1^{-1})^{1/2}=\sqrt{2m}$. In the absence of tunneling, the field ϕ is a free chiral right-propagating bosonic field. When L_t is nonvanishing, ϕ undergoes successive scattering at the points $x_{1,2}$ which breaks the charge conservation and creates a tunneling current. The phases κ_j in Eq. (2) include the contributions from external magnetic flux Φ_{ex} and from the average electron numbers $N_{0,1}$ on the two sides of the interferometer: $\kappa_2-\kappa_1=2\pi(\Phi_{ex}/\Phi_0)+(N_0/\nu_0)-(N_1/\nu_1)]+\text{const} \equiv -\kappa$. The voltage V between the edges can be introduced as a shift of the incoming field: $\phi_0 - \sqrt{\nu_0}Vt$, which translates into the following shift of $\phi \to \phi - Vt/\lambda$.

A thermodynamic Bethe ansatz solution known[13] for one point contact with $\lambda^2=2m$ can be generalized[8] to the tunneling at two contacts by successive application of boundary S-matrices[14] to the bosonic field excitations: kinks, antikinks, and breathers. For charge transport, only transitions between the kinks and antikinks are important, and their S-matrices are
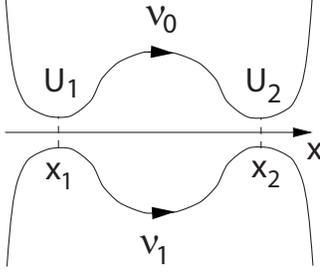
FIG. 1. Mach-Zehnder interferometer considered in this work: two contacts with tunneling amplitudes U_j formed at points x_j, between single-mode edges with different filling factors ν_0 and ν_1. The arrows show propagation direction of the edges.

$$\mathcal{S}_{j,k}^{\pm\pm} = \frac{(ak/T_{jB})^{m-1}e^{i\alpha_{j,k}}}{1+i(ak/T_{jB})^{m-1}}, \quad \mathcal{S}_{j,k}^{-+} = \frac{e^{i(\alpha_{j,k}-\kappa_j)}}{1+i(ak/T_{jB})^{m-1}}.$$

Here, the energy scales T_{jB} characterize the tunneling strength at the jth contact [explicit relation to the electron-tunneling amplitudes as follows from the perturbative calculations[8,13] is given next to Eq. (11)], and

$$a \equiv 2v\sqrt{\pi}\Gamma\{1/[2(1-\nu)]\}/(\nu\Gamma\{\nu/[2(1-\nu)]\}).$$

At zero temperature, kinks fill out all available "bulk" states with momentum k and distribution $\rho(k)$ up to some momentum A defined by the applied voltage. Each kink with momentum k undergoes successive scattering at the two contacts described by the product of the two boundary S-matrices, independently of other quasiparticles. Therefore, the large-time t asymptotics of the cumulant-generating function $\ln P(\xi)$ (logarithm of the Fourier transform of the probability distribution of transferred charge, which we measure in units of electron charge $e=1$) is expressed as a sum of independent contributions $\ln p(k,\xi)$ from individual momentum states,

$$\ln P(\xi) = t\int_0^A dk\rho(k)\ln p(k,\xi), \tag{3}$$

where $p(k,\xi)$ is expressed as usual,

$$p(k,\xi) = 1 + \tau_C(k)(e^{i\xi}-1) \tag{4}$$

through the total transition probability $\tau_C(k)$ of kink with momentum k into antikink. With parametrization $(T_{jB}/a)^2 \equiv \exp\{\theta_j/(m-1)\}$, it can be written as

$$\tau_C(k) = |(\hat{\mathcal{S}}_2\hat{\mathcal{S}}_1)^{-,+}|^2 = B[\tau(\theta_2,k)-\tau(\theta_1,k)], \tag{5}$$

through the transition probability in one contact

$$\tau(\theta_j,k) = |\hat{\mathcal{S}}_j^{-,+}|^2 = [1+k^{2(m-1)}e^{-\theta_j}]^{-1},$$

and the factor characterizing interference,

$$B(T_{jB},\kappa) = |T_{1B}^{m-1}+T_{2B}^{m-1}e^{i\kappa}|^2/[T_{2B}^{2(m-1)}-T_{1B}^{2(m-1)}].$$

Below, we take $\theta_2 \geq \theta_1$, and write θ_j as $\theta_{2,1} = \bar{\theta}\pm\Delta\theta_0$ with dimensionless $\Delta\theta_0 \geq 0$, i.e., $\exp\{\Delta\theta_0\} = (T_{2B}/T_{1B})^{m-1}$.

The aim of our subsequent derivation is to find the cumulant-generating function $\ln P(\xi)$ in terms of the two generating functions $\ln P_S$ for charge transfer in a single contact found from the Bethe ansatz solution by Saleur and Weiss.[15] This derivation does not need the explicit form of $\rho(k)$ and A which can be found in Ref. 13. Following the approach for one contact, we first relate $\ln P(\xi)$ in Eq. (3) to the effective tunneling current. To do this, we introduce the generalized tunneling probability

$$\tau_C(u,k) \equiv [1+(\tau_C^{-1}(k)-1)e^{-u}]^{-1}, \tag{6}$$

defined so that with the substitution $\tau_C(k) \to \tau_C(u,k)$ in Eq. (4), one has the following identity:

$$(-i\partial_\xi)^j\ln p(k,\xi)|_{\xi=0} = (\partial_u)^{j-1}\tau_C(u,k),$$

which shows that the cumulant-generating function is indeed related to a tunneling current defined by $\tau_C(u,k)$,

$$\partial_{i\xi}\ln P(u,\xi)/t = \int_0^A dk\rho(k)\tau_C(u+i\xi,k) \equiv I(u+i\xi,V).$$

Substituting Eq. (5) into Eq. (6), one finds $\tau_C(u,k)$ as difference of the single contact transition probabilities $\tau(\bar{\theta}\pm\Delta\theta(u),k)$, where $\Delta\theta(u)>0$ is defined by the equation

$$\cosh\Delta\theta(u) = \cosh\Delta\theta_0[1+R(e^u-1)], \tag{7}$$

with $R \equiv B\tanh\Delta\theta_0$. Further substitution of $\tau_C(u,k)$ into the definition of $I(u,V)$ results in the formula

$$I(u,V) = \partial_u\Delta\theta(u)\sum \pm I_{1/m}[\bar{\theta}\mp\Delta\theta(u),V], \tag{8}$$

which gives the derivative of the generating function in terms of the tunneling current $I_{1/m}(\theta,V)$ in one single-point contact as found from the Bethe ansatz solution.[13] Finally, integration of Eq. (8) over u presents $\ln P(\xi)=\ln P(u,\xi)|_{u=0}$ as the sum of two generating functions $\ln P_S$ for charge transfer in a single contact,

$$\ln P(\xi) = \sum_{j=1,2}\ln P_S(V/T_{jB}, e^{(-1)^{j-1}(\Delta\theta(i\xi)-\Delta\theta_0)}), \tag{9}$$

defined[15] by the low- and high-voltage expansions,

$$\frac{\ln P_S(s,e^{i\xi})}{\sigma_0 Vt} = \sum_{n=1}^\infty \frac{c_n(m)}{mn}s^{2n(1/\nu-1)}(e^{in\xi}-1), \quad s<e^\Delta,$$

$$= i\nu\xi + \sum_{n=1}^\infty \frac{c_n(\nu)}{n}s^{2n(\nu-1)}(e^{-in\nu\xi}-1), \quad s>e^\Delta,$$

$$c_n(\nu) = (-1)^{n+1}\frac{\Gamma(\nu n+1)\Gamma(3/2)}{\Gamma(n+1)\Gamma(3/2+(\nu-1)n)}. \tag{10}$$

Here $e^\Delta = (\sqrt{\nu})^{\nu/(1-\nu)}\sqrt{1-\nu}$, and σ_0 is the conductance quantum. Although Eq. (9) seems to suggest that the charge transfer process in the MZI is divided into two independent processes associated with two contacts of the MZI, such a division is not complete. Each charge transfer in Eq. (10) triggers multiple transfers in two contacts as reflected in Eq. (7) for $\Delta\theta(u)$ which enters Eq. (9).

At small voltages, $V<T_{1,2B}e^\Delta$, the two low-voltage expansions in Eq. (9) can be combined as follows:

$$\frac{\ln P(\xi)}{Vt\sigma_0} = \sum_{n=1}^{\infty} \frac{c_n(m)}{mn} \sum_j (V/T_{jB})^{2n(m-1)}$$

$$\times ([\cosh(n\Delta\theta(i\xi))/\cosh(n\Delta\theta_0)] - 1). \quad (11)$$

One can use here the standard expression for $\cosh[n\Delta\theta(i\xi)]$ as a polynomial of $\cosh[\Delta\theta(i\xi)]$, which is, according to Eq. (7), a linear function of $e^{i\xi}$. This shows that, in general, the nth term in Eq. (11) contains transfers of all numbers of electrons up to n. When $T_{2B} \gg T_{1B}$, one finds $R \to 1$ in Eq. (7), and the transfer statistics [Eq. (11)] approaches that of one point contact,[15] specifically the contact with the largest electron-tunneling amplitude U_1, related to T_{1B} by $T_{jB} = 2D(\Gamma(m)/U_j)^{1/(m-1)}$. In this case, the nth term in the voltage expansion series of the statistics describes the transfer of exactly n electrons. In the lowest order in V, MZI statistics (11) reduces to the Poisson distribution, with the factor in front of $(e^{i\xi} - 1)$ equal to the average electron-tunneling current.[8]

At large voltages, $V > T_{1,2B}e^\Delta$, steps similar to those in the low-V case, give

$$\frac{\ln P(\xi)}{Vt\sigma_0} = \sum_{n=1}^{\infty} \frac{c_n(1/m)}{n2^{n/m}} \left[\sum_j \left(\frac{T_{jB}}{V}\right)^{2(m-1)} \right]^{n/m} \sum_{\pm} [1 + R](z - 1)$$

$$\pm ([1 + R(z-1)]^2 - \cosh^{-2}\Delta\theta_0)^{1/2}]^{n/m}|_{z=1}^{z=e^{i\xi}}. \quad (12)$$

Again, for $T_{2B} \gg T_{1B}$, transfer statistics (12) coincides with that of one point contact,[15] but now with the largest quasiparticle tunneling amplitude W_2 related to T_{2B}, by $T_{jB} = 2mD(W_j/\Gamma(1/m))^{m/(m-1)}$. The nth term in expansion (12) corresponds to transfer of in general fractional charge n/m by n-quasiparticle. However, the large-voltage asymptotics (the $n=1$ term) of generating function (12) cannot be interpreted as a Poisson process of the lowest-order tunneling of individual quasiparticles of charge $1/m$. The reason for this is that the tunneling of each quasiparticle changes the interference phase by the statistical contribution $2\pi/m$ (Ref. 16) and therefore the tunneling rate of the next quasiparticle.

The appropriate description of such a dynamics of m phase states with different tunneling rates γ_l should be based, as usual, on the kinetic equation for probabilities to find MZI in one of the states, $l = 0, \ldots, m-1$. The equation can be written conveniently[17] in terms of the m dimensional vector function $Q_l(z,t) = \sum_n d_{l,n} z^{n/m}$, where $z = e^{i\xi}$, and $d_{l,n}$ are the probabilities to have MZI in state l and n tunneled quasiparticles,

$$\partial_t Q_l(z,t) = \sum_k M(z)_{l,k} Q_k(z,t), \quad (13)$$

where the transition matrix $M(z)_{l,k}$ is

$$M(z)_{l,k} = -\gamma_l \delta_{l,k} + \gamma_k z^{1/m} \delta_{l,k-1}, \quad (14)$$

with the Kronecker symbol $\delta_{l,k}$ defined modulo m. In what follows, we show that the leading large-V term of generating function (12) gives the same quasiparticle tunneling statistics as the kinetic equation (13). By doing this, we also extend this equation to the situation of different edge filling factors, when, as shown below, the quasiparticle exchange statistics

creates an additional shift $(m-1)\pi/m$ of the common interference phase.

We note first that the large-V limit of Eq. (9) is expressed through the quasiparticle amplitudes W_j as

$$\ln P(z) = tK[2W_1 W_2 \cosh(\Delta\theta(z)/m) - W_1^2 - W_2^2], \quad (15)$$

where $K \equiv \sigma_0 V(2mD/V)^{2(m-1)/m} c_1(1/m)/\Gamma^2(1/m)$. On the other hand, kinetic equation (13) gives[18] the same generating function as $\ln P(z) = t\Lambda$, where Λ is the maximum eigenvalue of transfer matrix (14), i.e., the solution that goes to zero at $z \to 1$ of the equation

$$\prod_l (\gamma_l + \Lambda) - z\prod_l \gamma_l = 0. \quad (16)$$

To see the equivalence of the two results we look for a general solution of Eq. (16) with the tunneling rates γ_l in the form $\gamma_l = K|W_1 + W_2 \exp\{i\phi_l\}|^2$, where $\phi_l = \phi + 2\pi l/m$ with some unknown ϕ. Substituting these expressions into Eq. (16) and using the two identities,

$$x^m - 1 = \prod_l (x - e^{i2\pi l/m}),$$

$$2^{m-1}\prod_l \left[\cos\phi_l + \cosh\left(\frac{\Delta\theta}{m}\right)\right] = [\cosh\Delta\theta - (-)^m\cos(m\phi)],$$

$$(17)$$

where the second follows from the first, one confirms that $\Lambda = \ln P(z)/t$ defined by Eq. (15) solves Eq. (16) if

$$\cosh\Delta\theta = (-)^m\cos(m\phi) + \frac{z}{2}\prod_l \frac{\gamma_l/K}{W_1 W_2}.$$

Calculating the RHS of this equation again with the help of Eq. (17), one can see that it gives precisely the definition of $\cosh\Delta\theta(z)$ Eq. (7), if ϕ is taken as

$$\phi = \kappa/m + (m-1)\pi/m. \quad (18)$$

This equation shows that besides the statistical shift $2\pi/m$ of the interference phase between the MZI states l and $l+1$, the quasiparticle exchange statistics also shifts the common phase ϕ from the externally defined value κ/m. This deviation of ϕ from κ/m is not important if m is odd. However, for even m this shift changes the "spectrum" of the interference phases ϕ_l. This ensures that there is no shift of the interference pattern of the tunnel current, e.g., its Fano factor (see below), between electron and quasiparticle regimes.

So far, we have established the dynamics of electron and quasiparticle tunneling that underlies the respective small- and large-voltage limits of the charge transfer statistics. Now we focus on the full range of the voltage dependence of the cumulants associated with this statistics to study the crossover between the two asymptotic regimes. The cumulants are defined by Eq. (8) as

$$\langle N^j(t)\rangle_c/t = \partial_u^{j-1} I(u,V)|_{u=0}. \quad (19)$$

The first cumulant, $I \equiv I(0,V) = \langle N(t)\rangle/t$, is the average tunneling current I, while the second cumulant is the spectral density of the current fluctuations at zero frequency $S_I(0) \equiv \langle N^2(t)\rangle_c/t$, which at zero temperature reflects the shot noise

associated with charge transfer. Equation (19) gives for this cumulant:

$$S_I(0) = (1 - B \coth \Delta \theta_0)I - B^2 \sum_{j=1,2} \partial_\theta I_{1/m}(\theta_j, V). \quad (20)$$

The Fano factor $F = S_I(0)/I$ reflects both the charge and statistics of the tunneling excitations and illustrates the transition between the electron and quasiparticle regimes in MZI. In the low-voltage limit, $F = 1$ as a result of the regular Poisson electron tunneling. In the quasiparticle, large-voltage, limit, we have from Eq. (20),

$$F = 1 - \frac{|W_1^m + W_2^m e^{i\kappa}|^2}{W_2^{2m} - W_1^{2m}} \left\{ \frac{W_2^{2m} + W_1^{2m}}{W_2^{2m} - W_1^{2m}} - \frac{1}{m} \frac{W_2^2 + W_1^2}{W_2^2 - W_1^2} \right\}.$$

This Fano factor corresponds to the dynamics of quasiparticle tunneling as described by kinetic equation (13). Because of the complex nature of this dynamics characterized by m different rates γ_l, F is not equal simply to the quasiparticle charge $1/m$ but varies as a function of parameters between $1/m$ and 1. This variation is illustrated most clearly by the case $m = 2$, when

$$F = 1 - |W_1^2 + W_2^2 e^{i\kappa}|^2/[2(W_1^2 + W_2^2)^2].$$

For $\kappa = 0$, phase shift (18) is such that the two quasiparticle tunneling rates coincide, $\gamma_1 = \gamma_2$, regardless of the ratio of the amplitudes W_j. Then $F = 1/2$ demonstrating explicitly the quasiparticle charge 1/2. Even in the quasiparticle regime, F reduces to electron value 1, if $W_1 \simeq W_2$ and $\kappa = \pi$. In this case, one of the rates γ is much smaller than the other, and on the large-time scale set by the smaller rate, the quasiparticles effectively tunnel together, restoring F back to 1.

For larger m, the Fano factor F can be calculated numerically. Figure 2 shows F for $m = 3$, e.g., for tunneling between the two $\nu = 1/3$ edges. The curves are shown for different degrees of asymmetry between the two contacts, and for two values of the interference phase: $\kappa = 0$ and $\kappa = \pi$. Similarly to $m = 2$, destructive interference drives F to its electron value 1 at all voltages. The transition region between electrons and quasiparticles is indicated by the minimum of the Fano factor, with F decreasing even below the "pure" quasiparticle
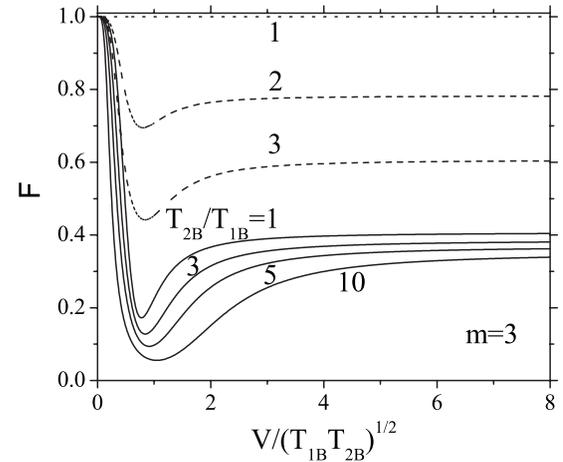

FIG. 2. The zero-temperature Fano factor F of the Mach-Zehnder interferometer formed by two $\nu = 1/3$ edges, as a function of the bias voltage V for different degrees of asymmetry, T_{1B}/T_{2B}, of the two contacts. The solid/dashed lines show the two values of the interference phase: $\kappa = 0$ and $\kappa = \pi$, respectively. The curves illustrate the transition from the electron regime $F = 1$ at small voltages to the quasiparticle m-state tunneling dynamics at large voltages.

value $1/m$. For symmetric junctions, such small value of F can be understood as a result of screening of the charge transfer in one contact by the other contact.

In conclusion, starting from the exact solution of the tunneling model of symmetric Mach-Zehnder interferometer in the FQHE regime, we have calculated the statistics of the charge transfer between interferometer edges. The statistics shows the transition from electron tunneling at low voltages to tunneling of anyonic quasiparticles of fractional charge e/m and statistical angle $2\pi/m$ at large voltages. Electron tunneling is characterized by the standard Poisson process. Dynamics of quasiparticle tunneling is more complicated and reflects the existence of m degenerate phase states of the interferometer. Quasiparticle braiding statistics shifts the common interference phase by $(m-1)\pi/m$ in these states, and also changes the phase by $2\pi/m$ from state to state. Crossover between electrons and quasiparticles manifests itself as minimum of the Fano factor of the tunneling current.

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